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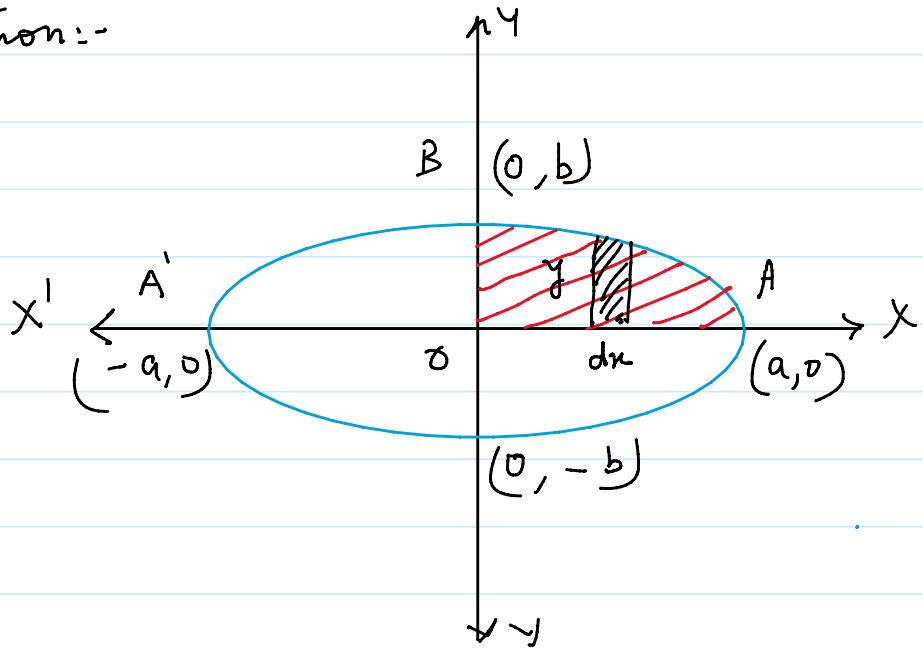
Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Solution:-



Area of the ellipse = 4 x area of OABO.

a ← x-axis value

$\therefore A = 4\int_0 y\,dx$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$

$\therefore y^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$\therefore y = \frac{b}{a}\sqrt{a^2 - x^2}$

$$\therefore y = \pm \frac{b}{a}\sqrt{a^2 - x^2}$$

Since region OAB O is in quadrant I, y is +ve

$$\therefore y = \frac{b}{a}\sqrt{b^2 - a^2}$$

$$\therefore A = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \frac{4b}{a}\left[\left(0 + \frac{a^2}{2}\sin^{-1}1\right) - (0 + 0)\right]$$

$$= \frac{4b}{a} \cdot \frac{a^2}{2} \times \frac{\pi}{2}$$

$$= \pi a b$$

$\therefore$ Area of ellipse $= \pi a b$.

For Your Practice:

① Find the area of the region bounded by the ellipse $\frac{x^2}{11} + \frac{y^2}{9} = 1$

ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(2) Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.