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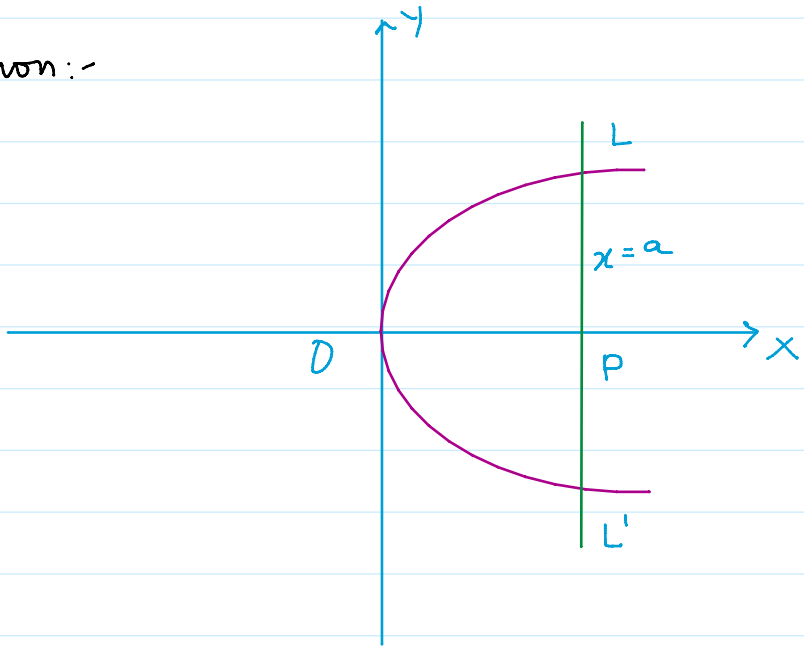
Calculate the area bounded by the parabola $y^2 = 4ax$ and its latus rectum.

Solution:-



Let $P(a,0)$ be the focus of the parabola $y^2 = 4ax$

The latus rectum LPL' is parallel to the y-axis & at a distance a from it.

∴ Equation of latus rectum is $x = a$.

Area is area of region $LOL'L$.

$$= \text{area of } LOPL + \text{area of } POL'P$$

$$= 2 \times \text{area}(LOPL)$$

$$\therefore \text{Area} = 2 \times \int_0^a y\,dx = 2\int_0^a 2\sqrt{ax}\,dx$$

$$\therefore \text{Area} = 4\sqrt{a}\int_0^a \sqrt{x}\,dx$$

$$\therefore A = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^a$$

$$= 4\sqrt{a}\left[\frac{2}{3}a^{3/2} - 0\right]$$

$$= 4\sqrt{a} \times \frac{2}{3} a^{3/2}$$

$$\therefore A = \frac{8}{3}a^2 \text{ sq. units}$$

— x —